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Spectral Domain Analysis of Dominant and Higher Order Modes in Fin-Lines

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***Abstract*—The spectral domain analysis is applied for deriving dispersion characteristics of dominant and higher order modes in fin-line structures. In addition to the propagation constant, the characteristic impedance is calculated based on the power–voltage definition. Numerical results are compared for different choices of basis functions and allow to estimate the accuracy of the solution.**

I. Introduction

THE FIN-LINE structure is a special printed transmission line proposed for millimeter wave integrated circuits in 1973 by Meier [1]. Since then, a number of millimeter-wave components have been developed in the fin-line form (e.g., [2]). The single-mode range of frequency is relatively wide, as the fin-line somewhat resembles the ridged waveguide. Propagation characteristics of fin-line structures have been investigated by a number of workers such as Hofmann [3], Hoefer [4], [5], and Saad and Begemann [10]. In [3], which is based on Galerkin's method in the space domain, sinusoidal functions are used as expansion functions. Hence a comparatively large number of expansion functions is required to obtain accurate results and, in addition, relative convergence problems occur and have to be handled carefully. On the other hand, some engineering approximations are involved in the work in [4].

In the present paper, fin-line structures are analyzed using the spectral domain technique, which has been developed for the analysis of various printed transmission lines for microwave integrated circuits [6], [7]. In this method, the information on the propagation constant at a given frequency is extracted from algebraic equations that relate Fourier transforms of the currents on the fins to those of the electric field in the dielectric–air interface. These equations are *discrete* Fourier transforms[1] of coupled integral equations one would obtain if the formulation is done in the space domain. Obviously, algebraic equations are much easier to handle in numerical processing. In addition to standard features of the spectral domain technique, the present work contains the following provisions.

1) The accuracy of the method is checked by comparing results obtained from three different choices of basis functions. A convergence check is also performed by increasing the number of basis functions for one of these sets.

2) In addition, dispersion curves for higher order modes are presented. For practical applications, the knowledge of higher order modes is important, because often a single mode operation is required.

3) Another important quantity for design purposes is the characteristic impedance of the dominant mode. By applying a definition suitable to fin-line structures, useful results for the characteristic impedance could be obtained and are presented in this paper.

II. Formulation of the Eigenvalue Problem

Since the details of the spectral domain method itself have been reported in [6] and [7], only the key steps will be given here. The method of using alternative sets of basis functions for accuracy checks has recently been

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[1]Henceforth referred to as Fourier transform.

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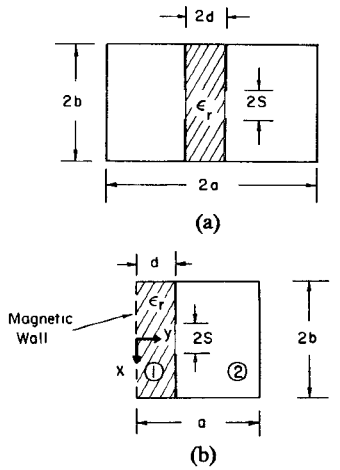

Fig. 1. Bilateral fin-line. (a) Cross section. (b) Equivalent structure.

employed for a higher order mode analysis of microstrip lines [8]. The significance of the modifications will be pointed out in this paper.

Several versions of fin-lines have been proposed, including bilateral, unilateral, and antipodal fin arrangements. Although the present method is applicable to other types of fin-line structures, we will formulate the problem for the bilateral fin-line, the cross section of which is shown in Fig. 1(a). Because of the symmetry, we only need to consider the half-structure given in Fig. 1(b).

Since the modal field in the fin-line is of hybrid type, the fields in regions 1 (dielectric) and 2 (air) can be derived from two scalar potentials $\phi_i(x, y)$ and $\psi_i(x, y)$, $i=1,2$, for instance

$$E_{zi}=j\frac{k_i^2-\beta^2}{\beta}\phi_i(x, y) \tag{1}$$

$$H_{zi}=j\frac{k_i^2-\beta^2}{\beta}\psi_i(x, y) \tag{2}$$

where $i=1,2$ signifies the region, k_i is the wavenumber in region i, and β is the propagation constant of the mode in the z direction. The time and z dependence of the field $\exp(j\omega t-j\beta z)$ is omitted throughout the paper. All other field components are derivable from Maxwell's equations.

In the spectral domain approach, the potentials ϕ_i and ψ_i as well as all the field quantities are Fourier transformed via

$$\tilde{\phi}_i(n, y)=\int_{-b}^{b}\phi_i(x, y)\exp\left(j\hat{k}_n x\right)dx \tag{3}$$

where $\hat{k}_n=n\pi/b$ for all odd (in E_z) modes including the dominant one and $\hat{k}_n=(n-1/2)\pi/b$ for the even modes. $\tilde{\phi}_i(n, y)$ and $\tilde{\psi}_i(n, y)$ now satisfy Helmholtz equations, e.g.,

$$\left(\frac{d^2}{dy^2}-\gamma_i^2\right)\tilde{\phi}_i(n, y)=0 \tag{4}$$

where

$$\gamma_1=\sqrt{\hat{k}_n^2+\beta^2-\epsilon_r k_0^2} \qquad \gamma_2=\sqrt{\hat{k}_n^2+\beta^2-k_0^2}$$

and k_0 is the free space wavenumber. Since the tangential electric fields E_{z2} and E_{x2} must be zero at $y=a, |x|<b$, and the tangential magnetic fields H_{z1} and H_{x1} be zero at $y=0, |x|<b$, appropriate solutions to the above equations are

$$\tilde{\phi}_1(n, y)=A_n^e\cosh\gamma_1 y \qquad \tilde{\phi}_2(n, y)=B_n^e\sinh\gamma_2(a-y)$$

$$\tilde{\psi}_1(n, y)=A_n^h\sinh\gamma_1 y \qquad \tilde{\psi}_2(n, y)=B_n^h\cosh\gamma_2(a-y).$$

By applying the interface conditions at $y=d$, the unknown coefficients A_n^e, A_n^h, B_n^e, and B_n^h can be eliminated, and we obtain two coupled algebraic equations

$$Y_{xx}\tilde{E}_x+Y_{xz}\tilde{E}_z=j\omega\mu_0\tilde{J}_x \tag{5}$$

$$Y_{zx}\tilde{E}_x+Y_{zz}\tilde{E}_z=j\omega\mu_0\tilde{J}_z \tag{6}$$

where

$$Y_{xx}=(\epsilon_r k_0^2-\beta^2)\frac{\tanh\gamma_1 d}{\gamma_1}+(k_0^2-\beta^2)\frac{\coth\gamma_2 h}{\gamma_2} \tag{7}$$

$$Y_{xz}=Y_{zx}=\beta\hat{k}_n\left(\frac{\tanh\gamma_1 d}{\gamma_1}+\frac{\coth\gamma_2 h}{\gamma_2}\right) \tag{8}$$

$$Y_{zz}=(\epsilon_r k_0^2-\hat{k}_n^2)\frac{\tanh\gamma_1 d}{\gamma_1}+(k_0^2-\hat{k}_n^2)\frac{\coth\gamma_2 h}{\gamma_2} \tag{9}$$

$$h=a-d$$

all are known. $\tilde{E}_x$, $\tilde{E}_z$, and $\tilde{J}_x$, $\tilde{J}_z$ are Fourier transforms of the unknown tangential electric field in the gap ($y=d, |x|<s$) and the unknown current components on the fins ($y=d, s<|x|<b$). Up to this stage the method of analysis is exact. In the following we present a solution based on Galerkin's method.

To this end, the unknown aperture fields $\tilde{E}_x$ and $\tilde{E}_z$ are expanded in terms of known basis functions $\tilde{\xi}_i$, $\tilde{\eta}_i$

$$\tilde{E}_x(\hat{k}_n)=\sum_{i=1}^{M}c_i\tilde{\xi}_i(\hat{k}_n) \tag{10}$$

$$\tilde{E}_z(\hat{k}_n)=\sum_{j=1}^{N}d_j\tilde{\eta}_j(\hat{k}_n) \tag{11}$$

where $\tilde{\xi}_i$ and $\tilde{\eta}_j$ are Fourier transforms of $\xi_i(x)$ and $\eta_j(x)$, which are chosen to be zero except for $|x|<s$.

Now (10) and (11) are substituted into (5) and (6) and the inner products of the resulting equations with $\tilde{\xi}_i$ and $\tilde{\eta}_j$, respectively, are obtained. This results in a homogeneous matrix equation for the unknown expansion coefficients c_i and d_j

$$\sum_{i=1}^{M}K_{pi}^{xx}c_i+\sum_{j=1}^{N}K_{pj}^{xz}d_j=0, \qquad p=1,\cdots,M \tag{12}$$

$$\sum_{i=1}^{M}K_{qi}^{zx}c_i+\sum_{j=1}^{N}K_{qj}^{zz}d_j=0, \qquad q=1,\cdots,N \tag{13}$$

where

$$K_{pi}^{xx}=\sum_{n=0}^{\infty}\tilde{\xi}_p(\hat{k}_n)Y_{xx}(\beta, \hat{k}_n)\tilde{\xi}_i(\hat{k}_n) \tag{14a}$$

$$K_{pj}^{xz}=\sum_{n=0}^{\infty}\tilde{\xi}_p(\hat{k}_n)Y_{xz}(\beta, \hat{k}_n)\tilde{\eta}_j(\hat{k}_n) \tag{14b}$$

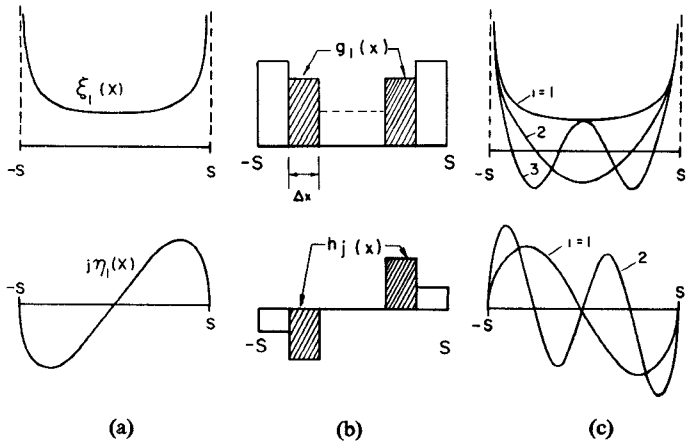

Fig. 2. Basis functions used for slot-field expansion (corresponding to (15)–(20)).

$$K_{qi}^{zx} = \sum_{n=0}^{\infty} \tilde{\eta}_q(\hat{k}_n) Y_{zx}(\beta, \hat{k}_n) \tilde{\xi}_i(\hat{k}_n) \tag{14c}$$

$$K_{qj}^{zz} = \sum_{n=0}^{\infty} \tilde{\eta}_q(\hat{k}_n) Y_{zz}(\beta, \hat{k}_n) \tilde{\eta}_j(\hat{k}_n). \tag{14d}$$

Equating the determinant of the coefficient matrix associated with (12) and (13) to zero, we finally obtain the eigenvalue equation, and its solutions are the desired propagation constants of the dominant and higher order modes.

III. Choices of Basis Functions

One of the features of the spectral domain method applied in the described manner is that quite accurate solutions result even if an extremely small size matrix such as $M=N=1$ is used. This takes place because certain qualitative natures such as the edge condition of the aperture electric field can be incorporated in the choice of basis functions.

In the present case three different choices of basis functions have been used, all of them being readily Fourier transformed analytically.

1) Qualified one-term expansions satisfying the edge condition at $|x|=s$ (see Fig. 2(a)):

$$\xi_1(x) = \frac{1}{\sqrt{s^2-x^2}} \tag{15}$$

$$\eta_1(x) = x \cdot \sqrt{s^2-x^2}\,. \tag{16}$$

2) Trains of rectangular pulses with unknown amplitudes (Fig. 2(b)):

$$\xi_i(x) = \begin{cases} 1, & (i-1)\cdot\Delta x < |x| < i\cdot\Delta x \\ 0, & \text{elsewhere} \end{cases} \tag{17}$$

$$\eta_j(x) = \begin{cases} 1, & (j-1)\cdot\Delta x' < x < j\cdot\Delta x' \\ -1, & -j\cdot\Delta x' < x < (j-1)\cdot\Delta x' \\ 0, & \text{elsewhere} \end{cases} \tag{18}$$

and $\Delta x = s/M$, $\Delta x' = s/N$.

3) Sinusoidal functions modified by an "edge condition" term (Fig. 2(c)):

$$\xi_i(x) = \frac{\cos\{(i-1)\pi(x/s+1)\}}{\sqrt{1-(x/s)^2}} \tag{19}$$

$$\eta_j(x) = \frac{\sin\{j\pi(x/s+1)\}}{\sqrt{1-(x/s)^2}}. \tag{20}$$

The second set of basis functions is numerically less advantageous than the others, because it requires an inherently larger matrix order, and the edge condition cannot be directly incorporated. However, it is very flexible and general and the expansion coefficients c_i, d_j are adjusted automatically to represent the aperture field distributions.

IV. The Characteristic Impedance

In addition to the propagation constants of the propagating modes, the characteristic impedance of the dominant mode is an important quantity for the design of microwave and millimeter-wave integrated circuits. Three definitions are possible for the impedance Z_c: they are V_x/I, $2P/I^2$, and $V_x^2/(2P)$ where P is the transmitted power, V_x the slot voltage, and I the current on the fins and waveguide walls. For reasons pointed out earlier [9], an impedance definition via slot voltage and transported power

$$Z_c = \frac{V_x^2}{(2P)} \tag{21}$$

was applied to the bilateral fin-line considered here. The calculation of the slot voltage

$$V_x = \int_{-s}^{+s} E_x(x, d)\,dx \tag{22}$$

can directly be accomplished, since the x dependence of the integral is known from the slot-field series expansion. The expansion coefficients c_i and d_j result from inverting the eigenvalue matrix equation, after the propagation constant has been computed. The transported power

$$P = \mathrm{Re}\int_{-b}^{+b}\int_{0}^{a} (E_x H_y^* - E_y H_x^*)\,dy\,dx \tag{23}$$

can be transformed to the spectral domain using Parseval's theorem

$$\tilde{P} = \mathrm{Re}\int_{0}^{a} (\tilde{E}_x \tilde{H}_y^* - \tilde{E}_y \tilde{H}_x^*)\,dy\,. \tag{24}$$

Since the spectral field components have already been derived in the course of formulating the eigenvalue problem, this integral can directly be solved and, thus, the calculation of the characteristic impedance completed.

It should be noted that the impedance defined in (21)–(24) is for one half of the bilateral fin-line in Fig. 1(b). The impedance thus defined may be called the one associated with one of the slots. The *total* impedance is one half of the values computed by the definition above, because the total power is twice that of (23); whereas, V_x given by (22)

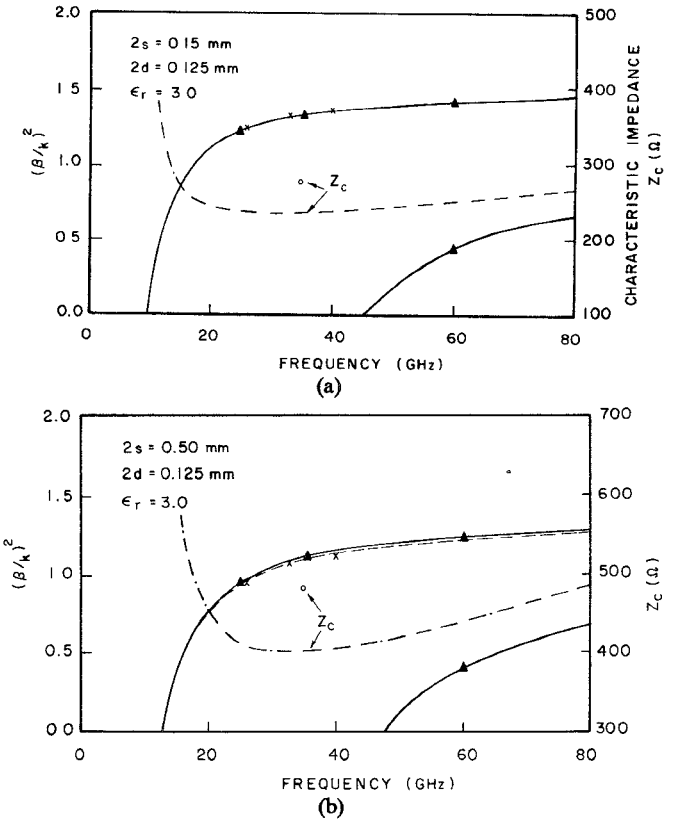

Fig. 3. Dispersion characteristics of dominant and first higher order mode and characteristic impedance of dominant mode. (200 spectral terms). (a) $2S=0.15$ mm. (b) $2S=0.50$ mm.——one-term expansion according to (15), (16), respectively, (19), (20), and $M=N=1$. ----- two-term expansion according to (19), (20) and $M=N=2$. (Indistinguishable from ——in (a)). ▲▲ expansion with rectangular pulses according to (17), (18) and $M=N=7$. ×× Hofmann's results [3]. ∘ Impedance by Hofmann [3]. –·– Characteristic impedance for $M=N=1$ and 2.

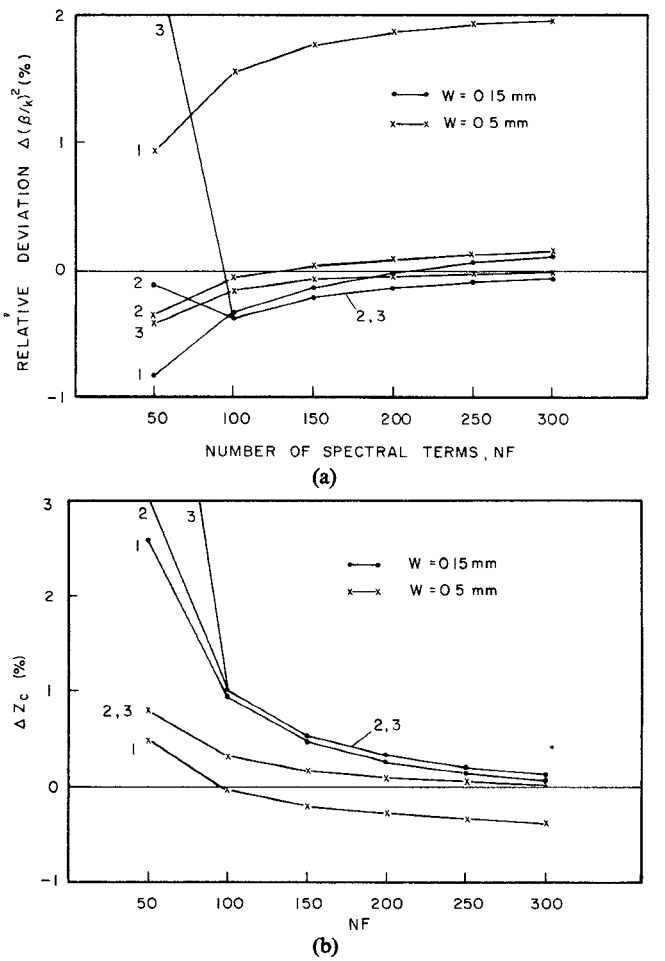

Fig. 4. Convergence behavior of (a) effective dielectric constant, and (b) characteristic impedance for $M=N=1,2,3$. (Expansion according to (19), (20); reference value: $NF=500$, $M=N=4$; $f=30$ GHz; $\epsilon_r=3$).

is identical to both slots. It is readily shown that total impedances based on other definitions are also one half of those for one slot.

V. Numerical Results

Dispersion characteristics for different choices of basis functions have been computed, including the effective dielectric constant $\epsilon_{eff}=\beta^2/k_0^2$ of the dominant and the first higher order mode and the characteristic impedance Z_c of the dominant mode (Fig. 3). Note that this definition of ϵ_{eff} is different from the one used in [1]. The latter provides information as to what kind of effects the dielectric substrate has in reference to the air-filled fin-line. Our present definition is simply square of the normalized propagation constant and hence, ϵ_{eff} here compares the guided wave in the structure with the plane wave in free space. Since the fin-line is not a quasi-TEM waveguide, the present ϵ_{eff} may be less than unity. This definition has also been used by Hofmann [3] and Jansen [9].

The dimensions of the shielding walls were chosen to coincide with the WR-28 waveguide usually utilized for 26.5 to 40-GHz operation. It is clearly seen that the dominant fin-line mode is not quasi-TEM but resembles somewhat the ridged waveguide dominant mode as pointed out earlier [1], [4].

Comparing the results for different sets of basis functions, a good conformity of all solutions can be testified for a small slotwidth. For broader slots (Fig. 3(b)), however, a one-term expansion results in relative errors of up to 2 percent. This fact is confirmed in Fig. 4, where the convergence behavior of ϵ_{eff} and Z_c is presented for one-, two-, and three-term expansions and a varying number of spectral terms. The solution for ϵ_{eff}, using a one-term expansion, obviously deviates about 2 percent from the reference value for a broad slot, whereas, the deviation of Z_c is not so significant. Further investigations have shown that for all normally used slotwidths, a two-term expansion according to (19), (20) and a number of 250 spectral terms are sufficient for very accurate solutions.

Fig. 5 shows modal dispersion characteristics for two more substrates and several slotwidths. In addition, in Fig. 5(b) the existence of modes resulting from an electric wall symmetry at $y=0$ is pointed out. Though modes with this kind of symmetry may not be excited by the dominant H_{10} mode of the empty waveguide, they have to be taken into account at all discontinuities that are not symmetric with respect to the $y=0$ plane.

Figs. 3 and 5 confirm Hofmann's statement in [3] that the characteristic impedance is fairly constant in the frequency range 30–40 GHz. Additionally these figures

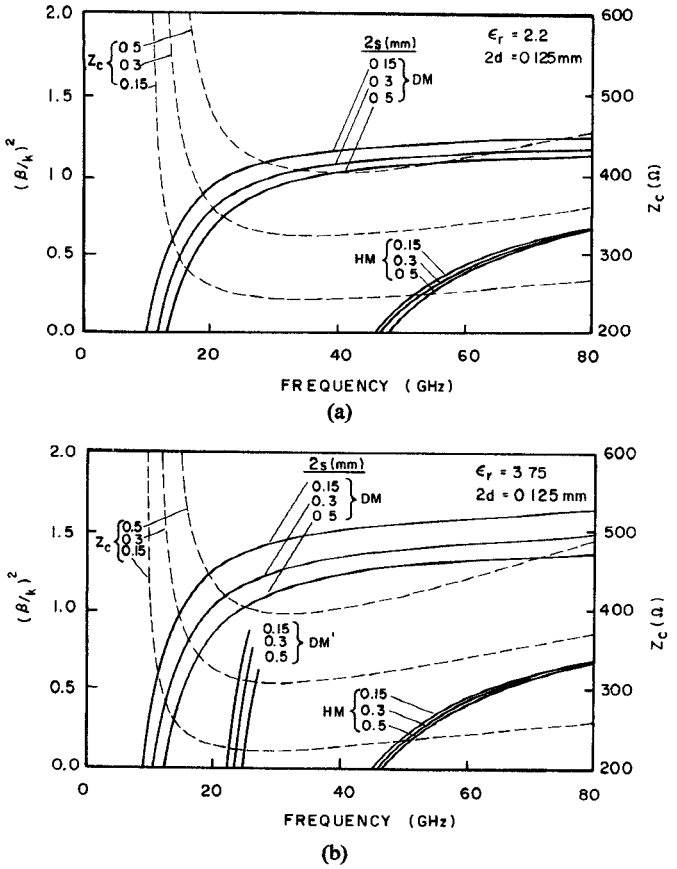

Fig. 5. Dispersion characteristics of effective dielectric constant and characteristic impedance for other substrates and different slotwidths $2s$. (*DM*-dominant mode, *HM*-higher order mode, *DM'*-dominant mode for the structure in Fig. 1(b) with magnetic wall replaced by an electric wall).

show a broad minimum of Z_c in this frequency range. Since we found that this phenomenon, which was not observed in Hofmann's work [3], is not due to numerical instability or convergence, we conjecture as follows. The main tendency of increasing impedance at higher frequencies can be interpreted with the help of the fact that the fields concentrate more and more in the vicinity of the slot as the frequency gets higher. This results in higher values of slot voltage V_x if the power P is set constant. Hence, $Z_c = V_x^2/(2P)$ becomes larger. Near the cutoff, however, another mechanism takes place. Since the power transmitted P in the z direction is zero at cutoff, Z_c becomes infinite. Since this cannot happen discontinuously, Z_c takes higher values again as the cutoff is approached, resulting in a broad minimum at intermediate frequencies. Since Hofmann [3] takes only the current on the fins into account in his definition $Z_c = V_x/I$, a different mechanism takes place for his impedance. At higher frequencies, the fin current increases as the fields concentrate near the slot, and the portion of current flowing on the waveguide walls decreases. Since the latter is neglected, Hofmann's impedance decreases at higher frequencies.

Hofmann's impedance values plotted in the figure are twice those found in his original paper. This is because he used the total impedance described earlier. Even after this factor of two is introduced, Hofmann's results deviate considerably from the present data. The deviations may be due to the fact that he used an impedance definition via slot voltage and fin current which seems to be questionable for broader slots, because the portion of the current on the waveguide wall which is neglected in his analysis becomes more important.

VI. Conclusions

Derived from a thorough dynamic spectral domain analysis, a wide variety of useful information about the bilateral fin-line has been given in this paper, including important aspects for practical application, as well as theoretical considerations about suitable choices of basis functions and the convergence behaviors of the solutions. Obviously, this method is applicable to other fin-line configurations as well.

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